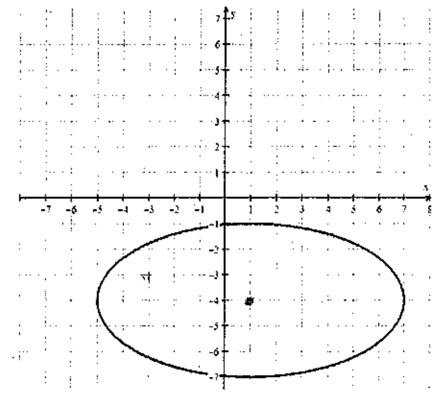
Find parametric equations for the ellipse shown on the right. **SCORE: _____ / 12 PTS**



$x = 1 + 6\cos t$

$y = -4 + 3\sin t$

Find $\dfrac{d^2y}{dx^2}$ at the point $(1, -4)$ on the curve with parametric equations $\begin{array}{l} x = t^3 - 9t + 11 \\ y = t^3 - 2t^2 - 4 \end{array}$. **SCORE: _____ / 24 PTS**

$\dfrac{dy}{dx} = \dfrac{3t^2 - 4t}{3t^2 - 9}$

$\dfrac{d^2y}{dx^2} = \dfrac{(6t-4)(3t^2-9) - (3t^2-4t)(6t)}{(3t^2-9)^3}$

$\left.\dfrac{d^2y}{dx^2}\right|_{t=2} = \dfrac{(8)(3) - (4)(12)}{3^3} = \dfrac{-24}{27} = -\dfrac{8}{9}$

$t^3 - 2t^2 - \cancel{4} = -\cancel{4}$

$t^2(t-2) = 0$

$t = \cancel{0}, 2$

$\downarrow \quad \downarrow$

$x \neq 11 \quad x = 1$

Write, **<u>BUT DO NOT EVALUATE</u>**, an integral (or sum of integrals) for the area of the shaded region, shown on the right, between the parametric curve $\begin{array}{l} x = 3 - (t+1)^3 \\ y = t^3 + t^2 - 2t \end{array}$ and the $x-$axis. **SCORE: _____ / 18 PTS**

$t^3 + t^2 - 2t = 0$

$t(t+2)(t-1) = 0$

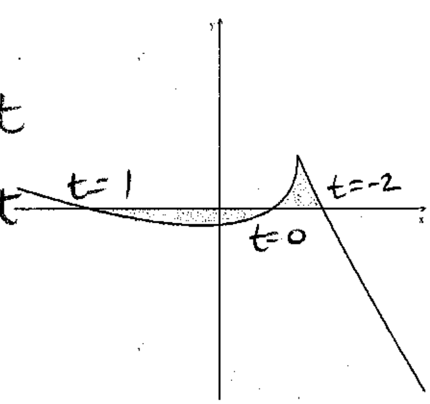
$t = -2, 0, 1$

$\downarrow \quad \downarrow \quad \downarrow$

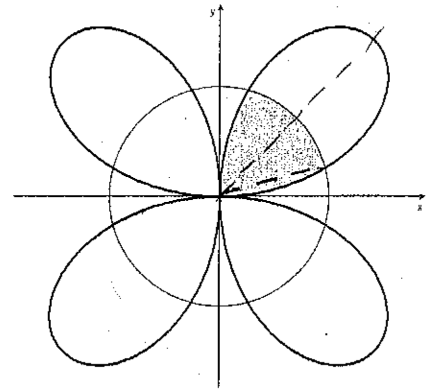
$x = 4 \quad x = 2 \quad x = -5$

$-\int_1^0 (t^3+t^2-2t)(-3(t+1)^2)\,dt$

$+\int_0^{-2} (t^3+t^2-2t)(-3(t+1)^2)\,dt$



Find the area of the shaded region inside both the polar curves $r = 2$ and $r = 4\sin 2\theta$ shown on the right. **SCORE: _____ / 24 PTS**



$4\sin 2\theta = 2$
$\sin 2\theta = \frac{1}{2}$
$2\theta = \frac{\pi}{6}, \frac{5\pi}{6}$
$\theta = \frac{\pi}{12}, \frac{5\pi}{12}$

$4\sin 2\theta = 0$
$\sin 2\theta = 0$
$2\theta = 0$
$\theta = 0$

$$2\left[\frac{1}{2}\int_0^{\frac{\pi}{12}} 16\sin^2 2\theta\, d\theta + \frac{1}{2}\int_{\frac{\pi}{12}}^{\frac{\pi}{4}} 4\, d\theta\right]$$

$$= \int_0^{\frac{\pi}{12}} (8 - 8\cos 4\theta)\, d\theta + 4\left(\frac{\pi}{4} - \frac{\pi}{12}\right)$$

$$= (8\theta - 2\sin 4\theta)\Big|_0^{\frac{\pi}{12}} + \frac{2\pi}{3}$$

$$= \frac{2\pi}{3} - \sqrt{3} + \frac{2\pi}{3}$$

$$= \frac{4\pi}{3} - \sqrt{3}$$

Find the **Cartesian coordinates** of all points on the polar curve $r = 1 - \sin\theta$ where the tangent line is vertical. **SCORE: _____ / 27 PTS**

$$\frac{dy}{dx} = \frac{-\cos\theta\sin\theta + (1-\sin\theta)\cos\theta}{-\cos^2\theta - (1-\sin\theta)\sin\theta} = \frac{\cos\theta - 2\sin\theta\cos\theta}{-(1-\sin^2\theta) - \sin\theta + \sin^2\theta}$$

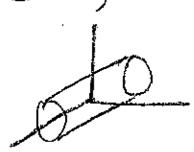
$$= \frac{\cos\theta(1-2\sin\theta)}{2\sin^2\theta - \sin\theta - 1}$$

$2\sin^2\theta - \sin\theta - 1 = 0$
$(2\sin\theta + 1)(\sin\theta - 1) = 0$
$\sin\theta = -\frac{1}{2}$ OR 1
$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$ OR $\frac{\pi}{2}$
$\downarrow \quad \downarrow \quad \downarrow$
$r = \frac{3}{2} \quad r = \frac{3}{2} \quad r = 0$

$\left(\frac{3}{2}\cos\frac{7\pi}{6}, \frac{3}{2}\sin\frac{7\pi}{6}\right) = \left(-\frac{3\sqrt{3}}{4}, -\frac{3}{4}\right)$
$\left(\frac{3}{2}\cos\frac{11\pi}{6}, \frac{3}{2}\sin\frac{11\pi}{6}\right) = \left(\frac{3\sqrt{3}}{4}, -\frac{3}{4}\right)$
AND $(0, 0)$

Describe in words the region in R^3 represented by the inequality $y^2 + z^2 \le 9$. **SCORE: _____ / 9 PTS**
Be as specific as possible. You may also sketch a diagram to clarify your answer.

SOLID CYLINDER, RADIUS 3, AXIS = X-AXIS



MULIPLE CHOICE [CIRCLE THE CORRECT ANSWER]: **SCORE: _____ / 6 PTS**

If $\vec{u} \cdot \vec{v} = -3$, then the angle between $\vec{u}$ and $\vec{v}$ **<u>might</u>** be

ANGLE IS OBTUSE

[a] 299° [b] 0° [c] 90° [d] 99° [e] 9°

Let ℓ_1 be the line with symmetric equations $x-3 = 4-y = \frac{z+1}{2}$. **SCORE: _____ / 15 PTS**

Let ℓ_2 be the line with parametric equations $x = 2t-3, \quad y = 4t+2, \quad z = 5-3t$.

Find the point-normal (ie. scalar) equation of the plane which is parallel to both ℓ_1 and ℓ_2 and passes through the point $(-1, 7, -2)$.

$$\vec{n} = \langle 1, -1, 2 \rangle \times \langle 2, 4, -3 \rangle$$
$$= \langle -5, 7, 6 \rangle$$
$$-5(x+1) + 7(y-7) + 6(z+2) = 0$$

Consider the vector expression $(\vec{a} \times \vec{b}) \times \vec{c}$. **SCORE: _____ / 15 PTS**

[a] Use the properties of the dot and cross products to rewrite the expression without using cross products. Simplify your answer so that none of the expressions $-\vec{a}, -\vec{b}$ nor $-\vec{c}$ appear.

$$-\vec{c} \times (\vec{a} \times \vec{b})$$
$$= (-\vec{c} \cdot \vec{b})\vec{a} - (-\vec{c} \cdot \vec{a})\vec{b}$$
$$= -(\vec{c} \cdot \vec{b})\vec{a} + (\vec{c} \cdot \vec{a})\vec{b}$$

[b] If $\vec{a}, \vec{b}$ and $\vec{c}$ are all unit vectors,
and $\vec{a}$ and $\vec{c}$ are orthogonal,
and the angle between $\vec{b}$ and $\vec{c}$ is 60°,
show that $(\vec{a} \times \vec{b}) \times \vec{c}$ is a vector of magnitude $\frac{1}{2}$ pointing in the opposite direction as $\vec{a}$.

$$-(\vec{c} \cdot \vec{b})\vec{a} + 0\vec{b}$$
$$= -(\|\vec{c}\| \|\vec{b}\| \cos 60^\circ)\vec{a} = -\tfrac{1}{2}\vec{a}$$